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Experimental Design of H_∞ Weighting Functions for Flight Control Systems

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This paper introduces an experimental solution for H_∞ weighting function selection by exploiting an experimental planning method that has been used in quality control. Conducting matrix experiments using special matrices, called orthogonal arrays, allows the effects of several weighting parameters to be determined efficiently so that the resulting H_∞ controller can satisfy many design specifications simultaneously in the environment for which the controller is designed. To show the feasibility and efficiency of this methodology, a flight control system for an airplane is designed to satisfy 11 performance specifications simultaneously, when the airplane is undergoing a large shift in c.g. position.

I. Introduction

BOTH H_∞ control theory[1–4] and μ synthesis[5] have become powerful design tools for handling large plant uncertainties to achieve stability and performance robustness. Many related software packages are now commercially available for synthesizing robust controllers based on the input specifications. Although H_∞ controllers are easy to obtain by using existing software packages,[6] most control engineers have encountered difficulty in choosing the qualified weighting functions. During the process of synthesizing H_∞ controllers for a flight control system, the most important and difficult procedure is to select appropriate weighting functions that can fully reflect the requirements of stability and performance.[7] Qualitative relations between design specifications and weighting functions exist for simple plants; however, for complex plants with stringent specifications, a systematic approach to determine the qualified weighting functions quantitatively is still lacking in the literature.

The purpose of this paper is to propose an experimental solution for weighting function selection by exploiting a robust design method[8] used in quality control. The original objective of robust design was to improve the quality of a product by minimizing the effect of process uncertainties without eliminating the causes. The tools used in robust design include the identification of a measure of quality loss and a set of experiments referred to as matrix experiments that will lead to a design that minimizes quality loss.

A matrix experiment consists of a set of experiments wherein we can change the settings of the parameters in a quality control activity which, for the present case, corresponds to the activity of choosing proper H_∞ weighting functions to ensure the satisfaction of stability and performance requirements. After conducting a matrix experiment, the data from all experiments in the set taken together are analyzed to determine the effects of the various parameters and to obtain the optimum setting of the parameters. Under certain conditions, conducting matrix experiments using special matrices, called orthogonal arrays, allows the effects of several weighting parameters to be determined efficiently. The main advantage of using a matrix experiment to determine controller parameters is its coherence and compliance with the practical environment within which the controller is designed to operate. When a matrix experiment is performed in a practical environment using real hardware, the experimental data are recorded and the optimum values of control parameters are determined in the presence of plant uncertainties such as sensor measurement error, saturation of actuators, the nonlinearities within plant dynamics, etc.

In this paper the H_∞ control theory and experimental planning method are combined to design a flight control system that satisfies 11 performance specifications and ensures robust stability when the airplane undergoes a wide range travel of c.g. positions. It is shown that an experimental planning method using orthogonal arrays tends to provide the optimal design of H_∞ weighting functions. Section II gives a brief discussion about the methodology being used and the underlying assumptions in matrix experiment. The flight control system to be designed and the specifications to be met are described in Sec. III and then the equivalent H_∞ control problem is formulated in Sec. IV. Finally, matrix experiments predicting the optimum H_∞ weighting functions are conducted in Sec. V.

II. Robust Design Using Matrix Experiments

An efficient way to study the effect of several parameters simultaneously is to plan matrix experiments. Matrix experiments are also called designed experiments; parameters are also called factors; and parameter settings are also called levels. It is shown in Ref. 8 that if a function for total quality loss Q_L can be formulated as a product of functions for quality loss due to the different parameters, then a reduced set of experiments can be formulated to cover all factors at all levels. This reduced set of experiments is referred to as an orthogonal array. Taguchi[9] has tabulated 18 basic orthogonal arrays. We refer the interested readers to Ref. 8 for more details about the procedures on constructing an orthogonal array to fit a specific case study.

To achieve additivity of the parameter effects, it is common practice to take the log of Q_L and express the result in decibels,

$$\eta = 10 \log_{10} (1/Q_L) \tag{1}$$

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Note that η is also called signal-to-noise ratio by Taguchi.[9] Since log is a monotonically increasing function, minimizing Q_L is equivalent to maximizing η. We will seek the optimum levels of each parameter with maximum η. To demonstrate the concept of robust design using a matrix experiment, we consider the following example. Assume there are four parameters A, B, C, and D to be determined, with each parameter having three levels. For example, A has three levels A_1, A_2, and A_3, B has three levels B_1, B_2, and B_3, etc. The purpose of the matrix experiment is to determine the best level for each parameter such that η is maximized. The traditional method would require 4^3 or 81 experiments. However, under the assumption that total quality loss can be formulated as a product of individual quality loss factors, Taguchi[9] has shown that the optimal solution can be obtained with only nine experiments.

The matrix experiment selected for this example is given in Table 1. It consists of nine individual experiments corresponding to the nine rows. The entries in the matrix represent the levels of the parameters. Thus, experiment 1 is to be conducted with each parameter at the first level.

After the values of η for each experiment are summarized, the next step is to estimate the effect of each parameter on the quality characteristics. First, the overall mean value of η is given by

$$m = \frac{1}{9}\sum_{i=1}^{9} \eta_i \tag{2}$$

By examining the columns in Table 1, we see that all of the three levels of every parameter are equally represented in the nine experiments. It can be shown that the columns of Table 1 are mutually orthogonal. Here, orthogonality is interpreted in the combinatoric sense; that is, for any pair of columns, all combinations of the parameter levels occur and they occur an equal number of times. This is called the balancing property. Thus, m is a balanced overall mean over the entire experimental region.

In fact, there is a strong analogy between the matrix experiment and the decomposition of the power of an electrical signal into different harmonics: 1) the nine observed values of η are analogous to the observed signal; 2) the sum of squared values of η is analogous to the power of the signal; 3) the overall mean m is analogous to the dc part of the signal; 4) the four parameters are like four harmonics; and 5) the columns in the matrix experiment are orthogonal, which is analogous to the orthogonality of the different harmonics.

The effect of a parameter level is defined as the deviation it causes from the overall mean. For example, we consider the effect of level A_1. It is observed that parameter A is at level 1 in experiments 1–3. The average η for these experiments, denoted by m_{A1}, is given by

$$m_{A_1} = ⅓\,(\eta_1 + \eta_2 + \eta_3) \tag{3a}$$

Thus, the effect of parameter A at level A_1 is given by $m_{A1} - m$. Average η for levels A_2 and A_3, as well as those for the various levels of the other parameters, can be obtained in a similar way. Hence, for example,

$$m_{B_2} = ⅓\,(\eta_2 + \eta_5 + \eta_8),\qquad m_{C_3} = ⅓\,(\eta_3 + \eta_5 + \eta_7) \tag{3b}$$

by noting that level B_2 appears in experiments 2, 5, and 8, and level C_3 appears in experiments 3, 5, and 7. The desired level for each parameter is the level that has the highest value of average η, i.e., has the highest effect.

At this stage, one may be concerned with the problem of whether the level with highest value of average η corresponds to the "optimum" level resulting in the minimum quality loss. The answer is affirmative provided the variation of η as a function of the factor levels follows the additive model described subsequently. An additive model is also referred to as a superposition model or a variables separable model in engineering literature. Note that superposition model implies that the total effect of several factors is equal to the sum of the individual factor effects. The relationship between Q_L and the parameters A, B, C, and D can be quite complicated. However, in most situations, when Q_L is chosen judiciously, η can be approximated adequately by the following additive model:

$$\eta(A_i, B_j, C_k, D_l) = m + a_i + b_j + c_k + d_l + e \tag{4}$$

In Eq. (4), m is the overall mean; the deviation from m caused by parameter A at level A_i is a_i; the terms b_j, c_k, and d_l represent similar deviations from m caused by the levels B_j, C_k, and D_l of parameters B, C, and D, respectively; and e stands for the error of the additive approximation and the error in the repeatability of measuring η for a given experiment. By definition a_1, a_2, and a_3 are the deviations from m caused by the three levels of parameter A. Thus,

$$a_1 + a_2 + a_3 = 0 \tag{5a}$$

similarly,

$$b_1 + b_2 + b_3 = c_1 + c_2 + c_3 = d_1 + d_2 + d_3 = 0 \tag{5b}$$

Under the additive model we can show that average η can be used to predict the optimum level, i.e., the level having the dominant effect. Consider Eq. (3b) for the estimation of the effect of level B_2

$$\begin{aligned} m_{B_2} &= ⅓(\eta_2 + \eta_5 + \eta_8) \\ &= ⅓(m + a_1 + b_2 + c_2 + d_2 + e_2) \\ &\quad + ⅓(m + a_2 + b_2 + c_3 + d_1 + e_5) \\ &\quad + ⅓(m + a_3 + b_2 + c_1 + d_3 + e_8) \\ &= ⅓(3m + 3b_2) + ⅓(a_1 + a_2 + a_3) \\ &\quad + ⅓(c_1 + c_2 + c_3) + ⅓(d_1 + d_2 + d_3) \\ &\quad + ⅓(e_2 + e_5 + e_8) \\ &= (m + b_2) + ⅓(e_2 + e_5 + e_8) \end{aligned}$$

Note that the terms corresponding to the effects of factors A, C, and D drop out because of Eq. (5). Thus, m_{B2} is an estimate of $m + b_2$; in other words, $m_{B2} - m$ is an estimate of the deviation from m caused by B_2. Now assume that A_2, B_1, C_3, and D_1 are the levels with the highest average value of η for parameters A, B, C, and D, respectively. The additive model, Eq. (4), can be used to predict the value of η under the optimum conditions just given.

$$\begin{aligned} \eta_{\text{opt}} &\approx m + a_2 + b_1 + c_3 + d_1 \\ &= m + (m_{A_2} - m) + (m_{B_1} - m) \\ &\quad + (m_{C_3} - m) + (m_{D_1} - m) \end{aligned} \tag{6}$$

Table 1 Demonstrated matrix experiment

Expt. no.	A	B	C	D	η dB
1	A_1	B_1	C_1	D_1	η_1
2	A_1	B_2	C_2	D_2	η_2
3	A_1	B_3	C_3	D_3	η_3
4	A_2	B_1	C_2	D_3	η_4
5	A_2	B_2	C_3	D_1	η_5
6	A_2	B_3	C_1	D_2	η_6
7	A_3	B_1	C_3	D_2	η_7
8	A_3	B_2	C_1	D_3	η_8
9	A_3	B_3	C_2	D_1	η_9

If the interactions within parameters are small compared to the main effects, then the observed response η_{obs} under the estimated optimum conditions will match the estimated η_{opt} based on the additive model. Furthermore, if additivity is indeed achieved, the matrix experiment provides simultaneously the global optimum levels over the discrete set we chose a priori. Fortunately, in most practical situations the additive model provides an excellent approximation. On the other hand, if additivity is not perfectly achieved, the estimated parameters are not optimum and the estimation error depends on the accuracy of additive model approximation; however, matrix experiments with orthogonal arrays still tend to result in well-performing parameters in practice. Selecting a good quality characteristic Q_L and parameters and their levels is essential in improving the additivity property. The selection process is not always easy. However, when experiments are conducted using orthogonal arrays, a verification experiment can be used to judge whether the interactions are severe.

In the present study, the primary goal in conducting a matrix experiment is to estimate the optimum levels for each parameter in the weighting functions to make the H_∞ controller most insensitive to the noise factor which, in the present case, is the airplane c.g. shift.

III. Flight Control System

An experimental aircraft developed by Aeronautical Industry Development Center (AIDC) is considered here. The longitudinal motion of the aircraft at the flight condition of 0.9 Mach number and 30,000-ft altitude is described by the following state-space model:

$$\begin{bmatrix}\dot\alpha\\ \dot u\\ \dot\theta\\ \dot q\end{bmatrix} = \begin{bmatrix}-3.313 & 0.0006 & 0 & 0.9032\\ -8.476 & -0.2596 & -34.37 & 0\\ 0 & 0 & 0 & 1.0000\\ 25.78 & -0.0060 & 0 & -3.022\end{bmatrix}\begin{bmatrix}\alpha\\ u\\ \theta\\ q\end{bmatrix} + \begin{bmatrix}-3.346 & -0.4579 & -0.1365\\ -2.485 & -40.77 & -98.85\\ 0 & 0 & 0\\ -65.53 & -2.451 & -15.34\end{bmatrix}\begin{bmatrix}\delta_H\\ \delta_{LEF}\\ \delta_{TEF}\end{bmatrix} \tag{7a}$$

$$\begin{bmatrix}N_z\\ q\end{bmatrix} = \begin{bmatrix}95.62 & -0.0103 & 0.002 & -0.4354\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\alpha\\ u\\ \theta\\ q\end{bmatrix} + \begin{bmatrix}-21.12 & 8.350 & -6.125\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\delta_H\\ \delta_{LEF}\\ \delta_{TEF}\end{bmatrix} \tag{7b}$$

where α, u, θ, q, and N_z stand for angle of attack (deg), velocity along the longitudinal axis (ft/s), pitch angle (deg), pitch rate (deg/s), and normal acceleration (ft/s^2), respectively. The proposed control configuration is depicted in Fig. 1. There are three control surfaces used in the longitudinal motion, namely, elevator (δ_H), leading-edge flap (δ_{LEF}), and trailing-edge flap (δ_{TEF}). The elevator deflection signal comes from the output of the H_∞ controller. The LEF/TEF deflections are trim controls and are scheduled with Mach number M and angle of attack α, providing the optimal lift/drag ratio. The bandwidth of surface actuator is 4 Hz. Two pilot prefilters are exploited to improve transient response. The normal acceleration N_z and pitch rate q are chosen as the feedback signals to facilitate the compatibility with the coupled flight/fire systems.

The main difficulty encountered in this control design is the large c.g. shift during the operation of the fire system. The c.g. position is related to static margin (SM),[10,11] which is defined

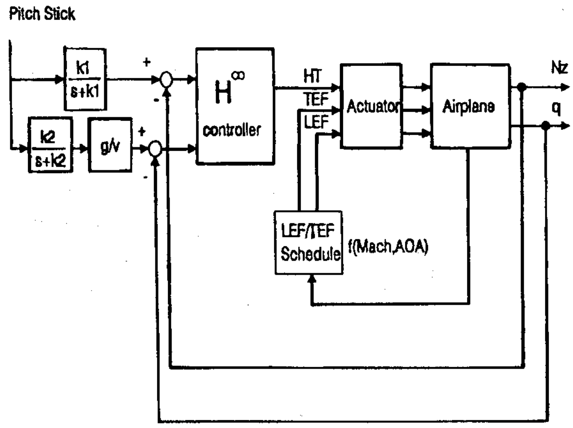


Fig. 1 Longitudinal control configuration.

as the distance between center of gravity X_{cg} and aerodynamic center X_{ac},

$$\mathrm{SM} = X_{cg} - X_{ac} \tag{8}$$

Static margin is an important index reflecting the relative stability of the airplane. The range of SM between the most forward and the most aft c.g. position of the present airplane is about 11%, leading to a significant change in low-frequency response as shown in Fig. 2. It can be observed that the elevator becomes much less effective in the forward c.g. position. As a consequence, it is difficult to apply conventional proportional plus integral (PI) controllers to meet the gain margin requirements for the whole operation range of SM.

The flight control system should be able to compensate for the effect of the SM shift and should meet the design specifications listed subsequently. Before proceeding further, some common quality characteristics for flight control system are defined. Based on MIL F-8785 C,[12] the short-period damping and natural frequency are calculated from the equivalent second-order system

$$\frac{\theta}{\theta_{com}} = \frac{k(s + 1/T_{\theta 2})e^{-\tau s}}{s^2 + 2\zeta_{sp}\omega_{sp}s + \omega_{sp}^2} \tag{9}$$

The original high-order system is fitted to this standard second-order response. The system parameters k, $T_{\theta 2}$, ζ_{sp}, ω_{sp}, and τ are estimated by minimizing the least-square error

$$\Sigma\{[\Delta\ \text{gain}]^2 + 0.017\ [\Delta\ \text{phase}]^2\} \tag{10}$$

where the model-matching errors of gain (dB) and phase (deg) are evaluated within the frequency range from 0.1 to 20 rad/s (Ref. 12). The design specifications are listed as follows:

1) The maximum allowable overshoot Q_{os} of pitch rate response is 35%.

2) The overshoot of N_z response should be zero.

3) The bandwidth ω_{bd} in pitch axis should lie between 6.5 and 11 rad/s.

4) The maximum allowable delay τ is 0.065 s. The allowable region spanned by the bandwidth ω_{bd} and time delay τ should be within the level-1 contour.[12]

5) The equivalent short-period damping ratio ζ_{sp} should be within the military level-1 limit, i.e., $0.4 \le \zeta_{sp} \le 1.4$ (Ref. 12).

6) The equivalent short-period natural frequency ω_{sp} should be within the military level-1 limit (Refs. 12 and 13).

7) The qualified drop-back time T_{db} (Ref. 14) for pitch control should be within the satisfactory area, i.e., $0 \le T_{db} \le 0.25$.

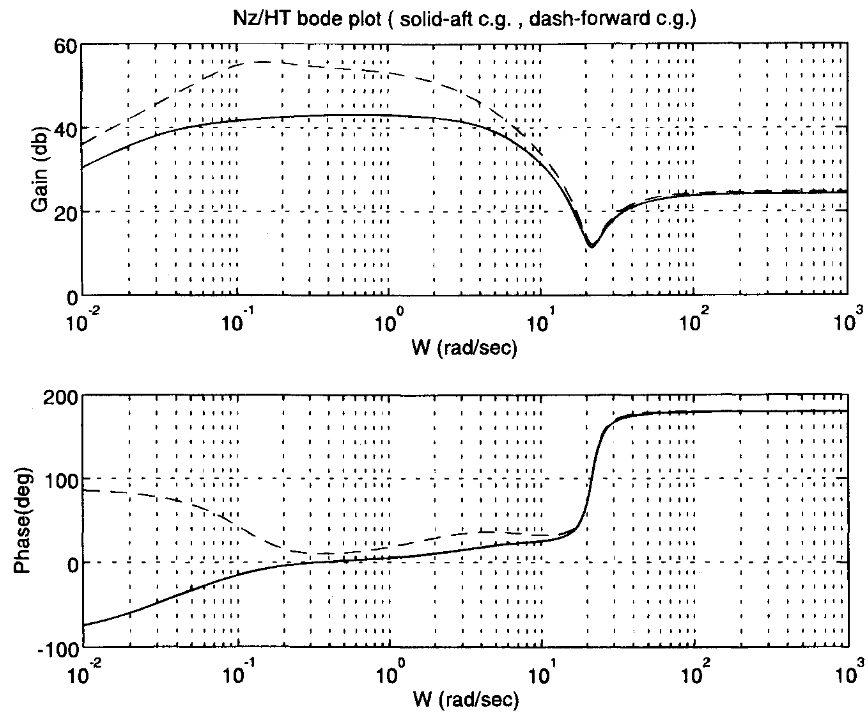


Fig. 2 Bode plot of transfer function from horizontal tail (HT) to normal acceleration (N_z).

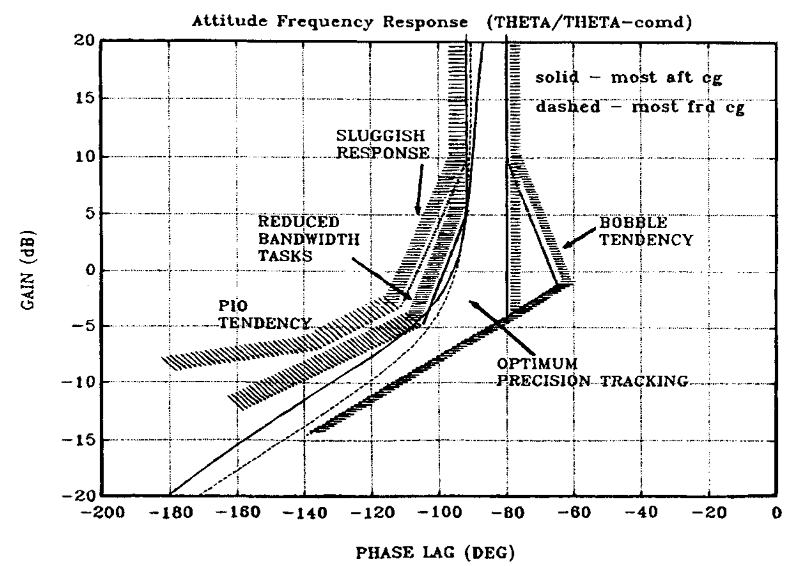


Fig. 3 Specifications of Gibson's frequency response.

8) The control anticipation parameter (CAP)[12] defined as

$$\mathrm{CAP} = [\omega_{sp}^2/(N_z/\alpha)] \tag{11}$$

should lie between 0.28 and 3.6 rad/s^2/g for fine tracking in longitudinal motion.

9) At least a 6-dB gain margin and a 30-deg phase margin are required in each surface break loop.[15]

10) The gain-phase curve of pitch-axis control loop should be within the area of optimal precision tracking in the Gibson's plot[14] as shown in Fig. 3.

11) The deflection magnitude and rate of elevator control surfaces should not exceed the limits ±25 deg and 70 deg/s, respectively.

IV. H_∞ Controllers and Weighting Functions

The flight control system shown in Fig. 1 can be recast into the standard feedback structure of Fig. 4 where r, d, and n represent reference command, disturbance, and measurement noise, respectively. There are two outputs to be controlled, i.e., the weighted tracking error z_1 and the weighted closed-loop response z_3. The measurement outputs are $y = [N_z \; q]^T$. The H_∞ controller $K(s)$ is to be determined to satisfy the H_∞ inequality,

$$\left\| \begin{matrix} W_1(s)S(s) \\ W_3(s)T(s) \end{matrix} \right\|_\infty < 1 \tag{12}$$

where $S(s) = [I + G(s)K(s)]^{-1}$ is the sensitivity function and $T(s) = G(s)K(s)\,[I + G(s)K(s)]^{-1}$ is the closed-loop transfer

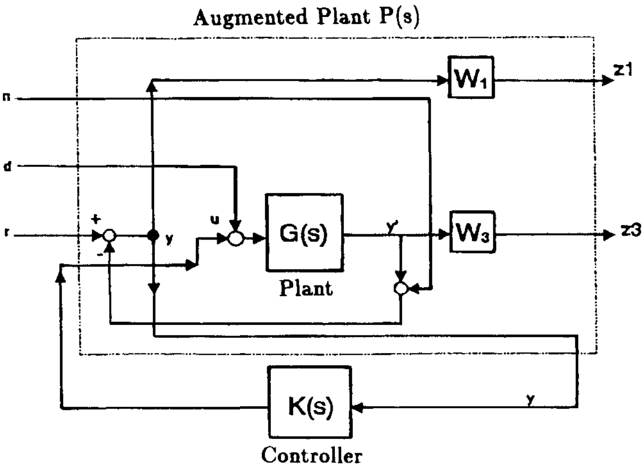


Fig. 4 Standard feedback control structure.

function from the reference command r to the measurement output y (also called the complementary sensitivity function). The weighting function $W_1(s)$ is incorporated intentionally to shape tracking-error distribution; whereas $W_3(s)$ is to shape the plant uncertainty caused by the c.g. shift. Because of the robust property of H_∞ controllers, it is straightforward to find an H_∞ controller to stabilize all of the plants characterized by all possible c.g. locations of the airplane. But it is very difficult to meet the 11 performance specifications in addition to the stability requirement. The difficulty arises in the selection of the appropriate weighting functions $W_1(s)$ and $W_3(s)$ to satisfy so many specifications simultaneously and to ensure robust stability with such a large c.g. shift. There exist some qualitative relations between weighting functions and closed-loop performance, but these relations can at most suggest a rough range of each parameter in weighting functions, and a large amount of effort is still needed to determine the specific numerical value of each parameter to meet all of the specifications.

This design difficulty can be conquered by matrix experiments. Matrix experiments using orthogonal arrays provide a very efficient and systematic methodology to predict the optimum setting of H_∞ weighting parameters. In the quality control community, the fundamental function of a matrix experiment is to determine the parameters to make the system performance insensitive to uncertainties, the so-called robust performance function. Accordingly, we can say, as will be verified subsequently, that although the H_∞ controller provides a good architecture for stability robustness, the matrix experiment, that provides well-qualified H_∞ weighting parameters, helps the H_∞ controller to ensure performance robustness, including in the presence of c.g. shift. Strong analogy between the present approach, i.e., design of H_∞ weighting functions by matrix experiments, and μ synthesis has been observed during the study of this work. Further understanding of this analogy is under investigation.

The weighting functions W_1 and W_3 in Eq. (12) and the prefilters shown in Fig. 1 are chosen in the following way.

$$W_1(s) = A\,\frac{s/B + 1}{s + 5 \times 10^{-4}} \tag{13a}$$

$$W_3(s) = D\,\frac{s/C + 1}{s/10^5 + 1} \tag{13b}$$

with prefilters

$$\frac{E}{s + E}, \qquad \frac{F}{s + F} \tag{13c}$$

Ideally, $W_1(s)$ must have poles at $s = 0$ to ensure zero tracking error in the presence of step input. The term 5×10^{-4} in Eq. (13a) is introduced intentionally to avoid numerical singularity caused by MATLAB algorithms. For good tracking performance, sensitivity function $S(s)$ should, in general, exhibit low-gain property over the low-frequency range. Since $\|W_1(s)S(s)\|_\infty \leq 1$ [from Eq. (12)], we recognize that $W_1(s)$ in Eq. (13a) must behave as a low-pass filter. As to the choice of $W_3(s)$, the dc gain D of $W_3(s)$ should be large enough to envelope the multiplicative plant uncertainty caused by c.g. shift. At the same time, the high-pass property of $W_3(s)$ is required to achieve enough bandwidth for the closed-loop transfer function $T(s)$ by noting $\|W_3(s)T(s)\|_\infty \leq 1$. The bandwidth of $T(s)$ is dominated by the factor C. Consequently, there are six parameters be determined, namely, A, B, C, D, E, and F. In some rare cases where stability and performance specifications are complicated and even conflicting, second-order or other higher degree weighting functions are needed to provide enough freedom for performance tradeoff. No matter how many parameters are involved in the weighting functions, matrix experiments can be equally applied by exploiting appropriate orthogonal arrays.

Some qualitative properties of H_∞ weighting functions can be found in the literature[16–18] that are helpful in estimating rough range of weighting parameter before conducting the matrix experiment. This rough range is then divided into several levels, and the best level of each parameter is determined by the following matrix experiments.

V. Conducting Matrix Experiments

Focusing on the application of matrix experiments to flight controller design, we do not intend to describe the theory of matrix experiments in detail; however, the underlying concept of a matrix experiment using orthogonal arrays is embedded in the following construction process of qualified weighting functions. We refer interested readers to Ref. 8 for more details about the general theory of matrix experiments. The optimization strategy of weighting functions consists of seven steps (A–G).

A. Identify Noise Factors

A noise factor is defined as a factor that can cause the response to deviate from the target performance and lead to quality loss. Plant uncertainty, system parameter variation, sensor noise, and disturbance can all be considered as noise factors. Here we are mainly concerned with the quality loss due to airplane c.g. shift. To establish experimental data, flight performance should be recorded for different c.g. locations. We will use two sets of data corresponding to the responses at the most forward and the most aftward c.g. locations to represent the effects of c.g. shift.

B. Identify Quality Characteristics and Objective Function

According to the design specifications mentioned earlier, the quality characteristics to be observed during matrix experiments will include the following items: 1) pitch rate overshoot Q_{os}, 2) normal acceleration overshoot N_{zos}, 3) bandwidth in pitch axis response ω_{bd}, 4) time delay in pitch axis response τ_p, 5) short-period damping ratio ζ_{sp}, 6) short-period natural frequency ω_{sp}, 7) drop back time T_{db}, 8) control anticipation parameter CAP, 9) gain/phase margins, and 10) Gibson's frequency response.

We will assign an on-off function f_i to each quality characteristic. For example, f_1, which is associated with the pitch rate overshoot Q_{os}, is set to 1 if specification 1 is satisfied, i.e., if the pitch rate overshoot is lower than 35%; otherwise, f_1 is set to zero. The remaining functions f_2–f_{10} are defined in the same way. Instead of the two-stage on-off function, we can adopt other multistage functions to quantify the quality characteristics more precisely. The total quality of flight performance is measured by the objective function J defined as

$$J = 0.15f_1 + 0.15f_2 + 0.2f_3 + 0.15f_4 + 0.1f_5 + 0.1f_6 + 0.1f_7 + 0.05f_8 \tag{14}$$

where the quality characteristics of items 9 and 10 will be checked outside the experiments and are not included in J. The reason for excluding items 9 and 10 from the matrix experiment is their strong coupling effects with items 1–8. A preliminary study shows that the inclusion of items 9 and 10 will considerably degrade the additivity property. The weights in J reflect the relative importance of each individual quality characteristic with respect to the total quality and are of different numerical values, depending on the mission of airplane.

C. Identify Control Factors and Their Levels

The control factors are those parameters that can be specified freely by the designer. The more complex a process, the more control factors the designer has and vice versa. In the process of synthesizing H_∞ controllers, the control factors are the parameters in the weighting functions $W_1(s)$ and $W_3(s)$ and the prefilters as shown in Eq. (13). For other control structures such as PID and linear quadratic Gaussian (LQG), they have different parameters to be specified. For example, in PID structure there are three gains to be determined; in LQG structure there are two weighting (transfer) matrices to be determined. No matter what kind of control structure is used, a matrix experiment tends to optimize the desired parameters.

Instead of assigning a specific controller structure a priori, we can also assume a general form of controller,

$$K(s) = \frac{b_m s^m + \cdots + b_1 s + b_0}{s^n + a_n s^{n-1} + \cdots + a_2 s + a_1} \tag{15}$$

and leave the coefficients a_i and b_i to be determined automatically by the mechanism of the matrix experiment to satisfy the stability and performance specifications.

The control factors in the present study are the four parameters A, B, C, and D in the weighting functions W_1 and W_3 and the two parameters E and F in the prefilters. The range of each parameter is divided into five levels. Table 2 shows the numerical value for each level.

Before dividing the range of each parameter into several levels, a rough range for each parameter should be known in advance. This rough range can be estimated by qualitative properties of the H_∞ weighting functions. If the control designer does not have any idea about the magnitude of the parameters, a preliminary matrix experiment can help locate a rough range for each parameter. This can be done, for example, by assuming a very large range for A, say from 0 to 10,000, and then divide this range into five levels. Suppose that the first run of the matrix experiment gives the optimum level between 0 and 2000 for A, we can further divide this range into several subintervals and continue the experiment to obtain a more refined range for A. After two or three iterations of the process, a reasonably small range for each parameter can be identified and the final refinement of the parameters can proceed, as we will do here.

It is a rule of thumb that at the start of the experiment we select three levels and take the levels sufficiently far apart so that a wide region can be covered by the three levels to fully reflect the nonlinear relations between parameters and quality characteristics. Commonly, one of these levels is taken to be the initial operating condition. Note that we are interested in the nonlinearity, so taking the levels of parameters too close together is not very fruitful. If we take only two levels, curvature effects would be missed, whereas such an effect can be identified by selecting three levels.

D. Design Matrix Experiment

The matrix experiment selected for this study is given in Table 3. It consists of 25 individual experiments corresponding to the 25 rows. The entries of the second block in the table represent the levels of the parameters. Accordingly, experiment 1 is to be conducted with each parameter being at its first level. Referring to Table 3, we see that the parameter levels for experiment 1 are $A = 0.500$, $B = 5.00$, $C = 220$, $D = 0.050$, $E = 1.00$, and $F = 5.00$. The matrix experiment of Table 3 exploits the standard orthogonal array L_{25} of Taguchi.[9,19] It can be shown that the columns of this array are mutually orthogonal.

E. Conduct Matrix Experiments

All of the 25 experiments have been conducted in a flight simulator that implements the complete six-degree-of-freedom nonlinear equations of aircraft motion with real aerodynamic data from wind-tunnel tests and with the thrust model from real engine tests. The flight control system being adopted is shown in Fig. 1. The simulation environment has been kept the same for all of the 25 experiments, except for the numerical values of the four parameters in the H_∞ weighting functions and the two parameters in the prefilters. Each experiment corresponds to a specific set of parameters. After the completion of each experiment, eight quality characteristics will be recorded, namely, Q_{os}, N_{zos}, ω_{bd}, τ_p, ζ_{sp}, ω_{sp}, T_{bd}, and CAP. Since we are interested in the variation of the quality characteristics due to airplane c.g. shift, all of the 25 experiments will be repeated for different c.g. locations. For example, for experiment 1 with the most forward (most aft, respectively) c.g. location, the eight quality characteristics evaluated from the simulator response are $Q_{os} = 14.9$ (3.1), $N_{zos} = 2.2$ (3.6), $\omega_{bd} = 4.22$ (4.63), $\tau_p = 0.051$ (0.045), $\zeta_{sp} = 0.79$ (0.82), $\omega_{sp} = 5.62$ (11.51), $T_{bd} = 0.13$ (0.27), and CAP $= 0.31$ (1.26). The total quality J for experiment 1 is then found from the quantitative measure defined in Eq. (14); and the results are $J = 25$ for the foremost c.g. position and $J = 50$ for the aftmost c.g. position. In the same manner, the values of J for the remaining 24 experiments can be constructed, and the results are listed in the third block of Table 3. Once the values of J for all of the 25 experiments have been established, we can estimate the optimum level for each parameter which results in the smallest variation of quality characteristics due to c.g. shift.

F. Analyze Data and Estimate Optimum Levels

The first step in data analysis is to summarize the data for each experiment. The objective function J is calculated from the eight quality characteristics via the relation (14). The central idea of the matrix experiment is to predict the optimum setting of the control factors such that the quality loss is minimized. The quality loss Q_L in our present case is defined as

$$Q_L = \frac{1}{n}\left(\frac{1}{J_1^2} + \frac{1}{J_2^2} + \cdots + \frac{1}{J_n^2}\right) \tag{16}$$

where J_i is the value of the objective function J evaluated at the z^{ith} noise level. Here, only two noise levels are considered,

Table 2 Levels of each control factor

Parameter	Level 1	Level 2	Level 3	Level 4	Level 5
A	0.50	1.50	2.50	3.50	4.50
B	5.0	20.0	35.0	50.0	65.0
C	220.0	270.0	320.0	370.0	420.0
D	0.05	0.10	0.15	0.20	0.25
E	1.00	2.00	3.00	4.00	5.00
F	5.00	10.0	15.0	20.0	25.0

Table 3 Matrix experiments using L_{25} orthogonal arrays

Exp.	A	B	C	D	E	F	Objective function Aft c.g.	Objective function Fore c.g.	S/N η
1	1	1	1	1	1	1	25	50	30.00
2	1	2	2	2	2	2	49	79	35.40
3	1	3	3	3	3	3	49	80	35.43
4	1	4	4	4	4	4	49	80	35.43
5	1	5	5	5	5	5	34	80	32.92
6	2	1	2	3	4	5	49	80	35.43
7	2	2	3	4	5	1	25	69	30.43
8	2	3	4	5	1	2	69	80	37.37
9	2	4	5	1	2	3	90	35	33.28
10	2	5	1	2	3	4	65	75	36.84
11	3	1	3	5	2	4	49	100	35.88
12	3	2	4	1	3	5	35	45	31.84
13	3	3	5	2	4	1	60	69	36.13
14	3	4	1	3	5	2	70	75	37.19
15	3	5	2	4	1	3	90	55	36.44
16	4	1	4	2	5	3	49	100	35.88
17	4	2	5	3	1	4	65	25	30.37
18	4	3	1	4	2	5	65	35	32.79
19	4	4	2	5	3	1	15	60	26.27
20	4	5	3	1	4	2	60	60	35.56
21	5	1	5	4	3	2	50	79	35.53
22	5	2	1	5	4	3	75	75	37.5
23	5	3	2	1	5	4	35	35	30.88
24	5	4	3	2	1	5	15	25	25.20
25	5	5	4	3	2	1	44	44	32.87

i.e., the two different c.g. locations. From the definition for J we know $J_i \leq 100$ (%), and the equality occurs when all of the eight specifications are satisfied. As the value of J_i becomes smaller, the quality deviates more from the target performance, and the magnitude of Q_L becomes larger. Hence, we can consider Q_L as a quantitative measure for quality loss. From the viewpoint of J, we would like it to be as large as possible. Why do we take the reciprocal of a larger-the-better type characteristic and then treat $1/J^2$ as a smaller-the-better type characteristic? It has been shown in Ref. 8 that minimizing the mean-square reciprocal quality characteristic, $(1/n) \sum 1/J_i^2$, has the effect of minimizing performance variation and, at the same time, maintaining the quality at its target performance; however, if we try to maximize the mean-square quality characteristic, $(1/n) \sum J_i^2$, we would have an adverse effect.

The ratio η calculated from Eq. (1) is listed in the last block of Table 3. For example, in experiment 1 the quality loss Q_L is calculated as $Q_L = \frac{1}{2}[(1/25^2) + (1/50^2)] = 10^{-3}$, and then the ratio η for experiment 1 is $10 \log_{10} 1/10^{-3} = 30$.

After the values of η for each experiment are summarized, the next step is to estimate the effect of each parameter on the quality characteristics. First, the overall mean value of η for the experimental region defined by the factor levels in Table 2 is given by

$$m = \frac{1}{25} \sum_{i=1}^{25} \eta_i = 33.71 \tag{17}$$

By taking the numerical values of η listed in Table 3, the average η for each level of the five parameters can be obtained as in Table 4. For example,

Table 4 S/N ratio of each parameter level

Parameter	Level 1	Level 2	Level 3	Level 4	Level 5
A	33.836	34.670	34.495[a]	32.173	32.395
B	34.543	33.108	34.520	31.474	34.925[a]
C	34.863[a]	32.884	32.501	34.678	33.645
D	32.312	33.888	34.258[a]	34.123	33.988
E	31.876	34.034	33.180	36.011[a]	33.461
F	31.140	36.211[a]	35.706	33.879	31.634

[a]Maximum S/N ratio.

$$m_{A_1} = \tfrac{1}{5}(\eta_1 + \eta_2 + \eta_3 + \eta_4 + \eta_5)$$
$$= \tfrac{1}{5}(30.00 + 35.40 + 35.43 + 35.43 + 32.92)$$
$$= 33.836$$
$$m_{B_3} = \tfrac{1}{5}(\eta_3 + \eta_8 + \eta_{13} + \eta_{18} + \eta_{23}) = 34.520$$

etc.

The entries in Table 4 represent the separate effects of each parameter and are commonly called main effects. From Table 4 we can determine the optimum level for each parameter as the level that has the highest value of η (noted in the table). Thus, from Tables 2 and 4, the best A is $A_3 = 2.500$; the best B is $B_5 = 65.000$; the best C is $C_1 = 220.000$; the best D is $D_3 = 0.150$; the best E is $E_4 = 4.000$, and the best F is $F_2 = 10.000$.

The estimated best setting need not correspond to one of the rows in the matrix experiment. Typically, the value of η realized for the estimated best setting is better than the best among the rows of the matrix experiment. Based on the results of the matrix experiment, the estimated optimum weighting functions and prefilters become

$$W_1 = 2.5 \frac{s/65 + 1}{s + 5 \times 10^{-4}} \tag{18a}$$

$$W_3 = 0.15 \frac{s/220 + 1}{s/10^5 + 1} \tag{18b}$$

$$\frac{E}{s + E} = \frac{4}{s + 4} \tag{18c}$$

$$\frac{F}{s + F} = \frac{10}{s + 10} \tag{18d}$$

We use these estimated optimum weighting functions to generate the desired H_∞ controller.

G. Conduct Verification Experiment

The various quality characteristics recorded from the flight simulator, when using the obtained H_∞ controller and prefilters, are listed in Table 5.

Table 5 Optimal quality characteristics

c.g. position	Q_{os}	Nz_{os}	W_{bd}	τ_p	ζ_{sp}	W_{sp}	T_{db}	CAP	Gain/Phase margin	J
Aft	17.4	1.5	6.54	0.032	1.07	14.06	0.01	1.94	15.7/59.3	100
Forward	13.0	0.0	7.34	0.031	0.98	16.58	0.04	2.62	∞/61.6	75

The quality characteristics are verified at two c.g. locations, i.e., the most forward and the most aft positions. The observed quality loss Q_L is calculated from Eq. (16) as $Q_L = (1/J_1^2 + 1/J_2^2)/2 = (1/100^2 + 1/75^2)/2 = 1.389 \times 10^{-4}$ and the observed η becomes $\eta_{obs} = 10 \log 1/Q_L = 38.57$. On the other hand, the prediction of the optimum η based on the additive model Eq. (4) is obtained as

$$\eta_{opt} = m + a_3 + b_5 + c_1 + d_3 + e_4 + f_2$$
$$= m + (m_{A_3} - m) + (m_{B_5} - m) + (m_{C_1} - m)$$
$$+ (m_{D_3} - m) + (m_{E_4} - m) + (m_{F_2} - m) = 42.19$$

where $m = 33.71$ from Eq. (17) and m_{A_3}, m_{B_5}, m_{C_1}, m_{D_3}, m_{E_4}, and m_{F2} can be read from Table 4. The closeness between η_{obs} and η_{opt} shows that additive model is a good approximation, and the estimated combination of parameters $A_3B_5C_1D_3E_4F_2$ is very close to the optimum situation. In the following, we show, from three aspects, that the aforementioned 11 design specifications are nearly satisfied during the variation of c.g. locations. The only exception is the $N_{Z_{os}}$ requirement. A 1.5% overshoot is observed in the N_z response at the foremost c.g. position.

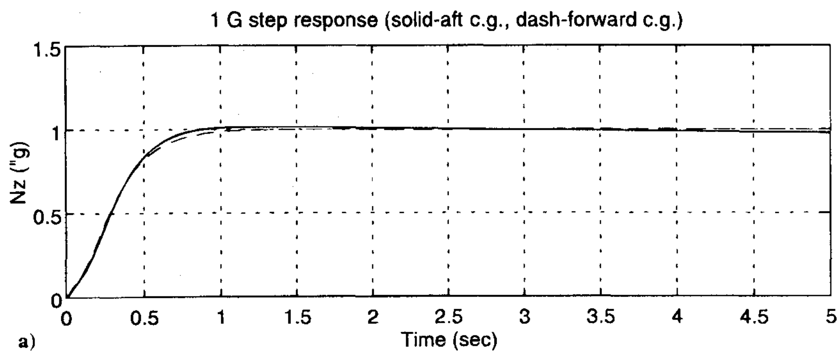
1) Frequency response: As can be seen from Table 5, the resulting gain margin is larger than 6 dB and the phase margin is larger than 45 deg for both c.g. locations. It can be checked that the relative magnitudes of gain/phase margin with respect to the bandwidth ω_{bd} for both c.g. positions satisfy military level-1 requirement. Referring to Fig. 3, we see that the gain-phase trajectory of pitch axis control loop for both c.g. locations fall within the satisfactory area of optimal tracking in the Gibson's plot.

2) Time response: In this part we check the satisfaction of the time responses for N_z and q. Figure 5a shows that no overshoot exhibits in the N_z response at the foremost c.g. position, although the overshoot at the aftmost c.g. position is 1.5%. This small overshoot is also indicated in Table 5 and is the only quality characteristic that does not meet the requirement of performance robustness. The overshoot in pitch rate response as shown in Fig. 5b does not exceed the 35% upper limit. The surface rate $\dot{H}_T$ and the surface deflection H_T are far below the saturation limit. The drop-back times T_{db} for both c.g. locations are between 0 and 0.25 as expected.

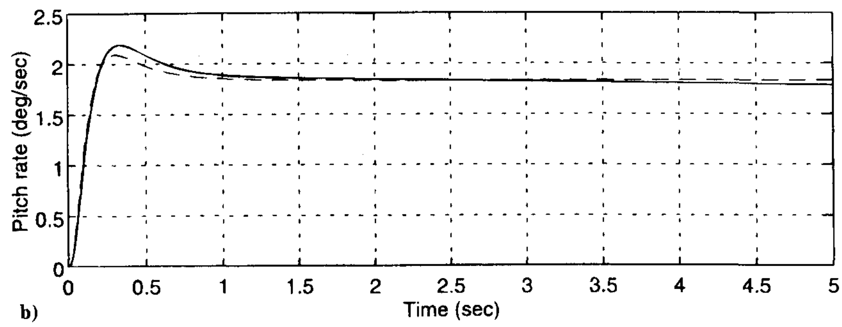
3) Equivalent system: As shown in Table 5, the short-period damping ratio ζ_{sp} is between 0.4 and 1.4 and the natural frequency ω_{sp} of the equivalent second-order system satisfies the level-1 requirements. Finally, the resulting control anticipation parameter CAP for both c.g. locations are between 0.28 and 3.6 as expected.

Although this controller does not meet the specifications precisely, its tendency of maintaining performance satisfaction in the presence of plant uncertainty is incontrovertible. Actually, H_∞ controllers satisfying precisely the requirement of performance robustness can be obtained by adopting second-order weighting functions instead of those in Eq. (13). In this case, we have five free parameters for each weighting function, and hence there are 10 parameters to be determined. The L_{50} matrix experiment is very suitable for this purpose if we still assign five levels to each parameter. However, it should be noted that precise performance robustness is achieved at the cost that the number of experiments (50) is now twice the previous one and the degree of the resulting H_∞ controller is increased by two.

As to stability robustness, since the weighting function $W_3(s)$ has been chosen to envelop the largest possible plant uncertainty (worst case) caused by the c.g. shift at the foremost and the aftmost positions, this $W_3(s)$ can also envelop the plant uncertainties over all intermediate c.g. positions. Consequently, the condition $\|W_3(s)T(s)\|_\infty < 1$ ensures that the aircraft is stabilized by the resulting H_∞ controller at any c.g. position. In summary,



a)



b)

Fig. 5 1G step response for a) normal acceleration N_z and b) pitch rate q.

the present approach exploits H_∞ controllers to provide stability robustness and exploits matrix experiments to provide parameter selection with performance robustness. It is worth noting that the tendency of the resulting H_∞ controller to achieve performance robustness and stability robustness is very similar to the inherent property of μ synthesis in spite of their totally different approaches. It may become an interesting research topic to relate present experimental design to its theoretical counterpart—μ synthesis.

VI. Conclusions

In this paper, we propose a new methodology of choosing H_∞ weighting functions by exploiting matrix experiments. It has been shown that matrix experiments using orthogonal arrays provide a very efficient and systematic way to estimate the optimum setting of the weighting parameters according to the prescribed stability and performance specifications, especially when the specifications are complex and conflicting. Of great importance is that the conclusions arrived at from the few experiments are valid over the entire experimental region spanned by the parameters and their levels. Hence, the required time and cost for the experiments can be reduced significantly. Finally, it is worth noting that under the structure of matrix experiment, the desired weighting functions and the resulting H_∞ controller can be determined using real hardware and in the environment within which the controller is designed to operate.

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References

[1]Doyle, J. C., Glover, K., Khargonekar, P. P., and Francis, B. A., "State-Space Solutions to Standard H_2 and H_∞ Control Problems," *IEEE Transactions on Automatic Control*, Vol. AC-34, 1989, pp. 831–847.

[2]Francis, B. A., and Doyle, J. C., "Linear Control Theory With an H_∞ Optimization Criterion," *SIAM Journal on Control and Optimization*, No. 25, 1987, pp. 815–820.

[3]Francis, B. A., *A Course in H_∞ Control, Lecture Notes in Control and Information Science*, Springer-Verlag, 1987.

[4]Maciejowski, J. M., *Multivariable Feedback Design*, Addison-Wesley, 1989.

[5]Stein, G., and Doyle, J. C., "Beyond Singular Values and Loop Shapes," *Journal of Guidance, Control, and Dynamics*, No. 14, 1991, pp. 5–16.

[6]Anon., *Robust Control Toolbox*, 386-Matlab User Guide, The Mathworks, Inc., 1989.

[7]Voulgaris, P., and Valavan, L., "High Performance Linear Quadratic and H_∞ Designs for a Super Maneuverable Airfcraft," *Journal of Guidance, Control, and Dynamics*, Vol. 14, No. 1, 1991, pp. 157–165.

[8]Phadke, M. S., *Quality Engineering Using Robust Design*, Prentice Hall, 1989.

[9]Taguchi, G., *System of Experimental Design*, Vols. 1 and 2, edited by Don Clausing, UNIPUB/Kraus International, New York, 1987.

[10]Etkin, B., *Dynamics of Atmospheric Flight*, Wiley, 1972.

[11]Roskam, J., *Airplane Flight Dynamics and Automatic Flight Controls*, Pts. I and II, 1979.

[12]Anon., "Military Specification, Flying Qualities of Piloted Airplane," MIL-F-8785C, Nov. 1980.

[13]Anon., "Equivalent System Verification and Evaluation of Augmentation Effects on Fighter Approach and Landing Flying Qualities," AFWAL-TR-81-3116, July 1981.

[14]Gibson, J. C., "Piloted Handling Qualities Design Criteria for High Order Flight Control Systems," AGARD FMP Conf., April 1982.

[15]Anon., "Military Specification, Flight Control Systems—Design, Installation and Test of Piloted Aircraft," MIL-F-9490D, USAF.

[16]Postlethwaite, I., Tsai, M. C., and Gu, D. W., "Weighting Function Selection in H_∞ Design," Proceedings of the IFAC Conf., Tallinn, Estonia.

[17]Tsai, M. C., Geddes, E. J. M., and Postlethwaite, I., "Pole-Zero Cancellations and Closed-loop Properties of an H_∞ Mixed Sensitivity Design Problem," *Automatica*, Vol. 28, 1992, pp. 519–530.

[18]Yeh, F. B., and Yang, C. D., *Post Modern Control Theory and Design*, Eurasia, 1991.

[19]Taguchi, G., "Off-Line and On-Line Quality Control System," International Conf. on Quality Control, Tokyo, Japan, 1978.